Unexpectedly small empirical vector strangeness of nucleons realized in a baryon model

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Abstract

Most of model considerations of the hidden nucleon strangeness, as well as some preliminary experimental evidence, led to the expectations of relatively sizeable strange vector form factors of the proton. For example, it seemed that the contribution of the fluctuating strange quark-antiquark pairs accounts for as much as one tenth of the proton's magnetic moment. By the same token, baryon models which failed to produce the "vector strangeness" of the nucleon seemed disfavored. Recently, however, more accurate measurements and more sophisticated data analysis, as well as lattice simulations, revealed that the form factors associated with the vector strangeness of the nucleon are much smaller than thought previously; in fact, due to the experimental uncertainties, the measured strange vector-current proton form factors may be consistent with zero. In the light of that, we re-asses the merit of the baryon models leading to little or no vector strangeness of the nucleon. It is done on the concrete example of the baryon model which essentially amounts to the MIT bag enriched by the diluted instanton liquid.

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1 Introduction

The simple, "naive" picture of hadrons, based on the models where only valence quarks are present, suffers a radical change when one takes into account the quantum field effects and consequently, the presence of the fluctuating virtual quark-antiquark ($q\bar{q}$) pairs. Indeed, the production of such fluctuating virtual pairs by interactions present in quantum chromodynamics (QCD) can be quite significant for the light quark flavors $q = u, d, s$.

The nucleon states, although they of course contain no *net* strangeness, are thus expected to have also an intrinsic strangeness component due to fluctuating $s\bar{s}$ pairs. Precisely because nucleons have no valence strange (s) quarks, quantities originating from, or being influenced by strange quarks, provide us with the information on the dynamics of virtual quarks within nucleons.

A review of the issue of nucleon strangeness containing a very complete set of original references is for example Ref. [1], and for more recent discussions of nucleon structure addressing also the nucleon strangeness issue, see for example Refs. [2, 3, 4].

Such considerations have led to the wide-spread belief that strange quarks and antiquarks play a major role in protons and neutrons. For example, although estimates of such $s\bar{s}$ contributions to the nucleon mass vary between 100 to 300 MeV, they are in any case very significant, between 10% to 30% of the nucleon mass. However, the quantities related to the present considerations are the proton magnetic moment and related electromagnetic form factors of the proton, so now we turn to them.

2 Strange form factors

The Dirac and the Pauli strange vector form factors (denoted by F_1^s and F_2^s, respectively) of the nucleon (N) are defined through the matrix element of

$$V_\mu^s = \bar{s}\gamma_\mu s \, , \tag{1}$$

namely the vector current of s-quarks:

$$\langle N|V_\mu^s|N\rangle = \langle N|\bar{s}\gamma_\mu s|N\rangle = \bar{u}_N(p')\left[F_1^s(q^2)\gamma_\mu + F_2^s(q^2)\frac{i\sigma_{\mu\nu}q^\nu}{2M_N}\right]u_N(p) \, , \tag{2}$$

where u_N is a nucleon spinor, p' and p are nucleon momenta, and $q = p' - p$ is the transferred momentum.

Although $F_1^s(0) = 0$, as it is the net nucleon strangeness, its momentum dependence determines the strangeness radius

$$r_s^2 = 6\frac{d}{dq^2}F_1^s(q^2)\bigg|_{q^2=0} \, , \tag{3}$$

while the strange magnetic moment is given by

$$\mu_s = F_2^s(0) \ . \tag{4}$$

For comparison with experimental data, the (strange) Sachs form factors G_E^s (electric) and G_M^s (magnetic) are widely used:

$$\begin{aligned} G_E^s(q^2) &= F_1^s(q^2) + \frac{q^2}{4M_N^2} F_2^s(q^2) \ , \\ G_M^s(q^2) &= F_1^s(q^2) + F_2^s(q^2) \ . \end{aligned} \tag{5}$$

Taking the non-relativistic nucleon spinor (of momentum p and spin projection ζ)

$$u_N(p,\zeta) = \sqrt{\frac{E+M_N}{2E}} \begin{pmatrix} \chi_\zeta \\ \frac{\boldsymbol{\sigma}\cdot\vec{p}}{E+m}\chi_\zeta \end{pmatrix} \ , \tag{6}$$

where χ_ζ is a two-component Pauli spinor. Going to the Breit frame defined by

$$\begin{aligned} q^\mu &= (q^0, \vec{q}) = (0, \vec{q}_B) \ , \\ \vec{p} &= \frac{\vec{q}_B}{2} \ , \ \vec{p}' = -\frac{\vec{q}_B}{2} \ , \end{aligned} \tag{7}$$

the components of the vector-current nucleon matrix elements are expressed by Sachs form factors through the relations

$$\langle N(p',\zeta')|V_0^s|N(p,\zeta)\rangle = \frac{m}{E}\chi_{\zeta'}^\dagger \chi_\zeta G_E^s(-\vec{q}_B^{\,2}) \ , \tag{8}$$

$$\langle N(p',\zeta')|\vec{V}^s|N(p,\zeta)\rangle = \frac{1}{2E}\chi_{\zeta'}^\dagger i(\boldsymbol{\sigma}\times\vec{q}_B)\chi_\zeta G_M^s(-\vec{q}_B^{\,2}) \ . \tag{9}$$

3 How we get vector strangeness

In the recent past, numerous model and lattice calculations gave very differing results for such strangeness contributions. For example, various results on the $s\bar{s}$ contribution μ_s to the proton magnetic moment range from 0.003 to as high as 0.8 nucleon magnetons (μ_N) in absolute magnitude. What is more, they differ among each other even up to a sign. (For overview and references, see Ref. [2].)

Overall, majority of the model calculations of nucleon strangeness led to the expectations of substantial strangeness contributions to the vector form factors and the magnetic moment of the nucleon. In contrast to that, the model introduced by Klabučar *et al.* [5] and elaborated in Refs. [6, 7], yields zero results for these strange quantities, although it reveals substantial scalar strangeness. This is in accord with the conjecture [8] that a nontrivial QCD-vacuum structure selects the pseudoscalar and scalar channels,

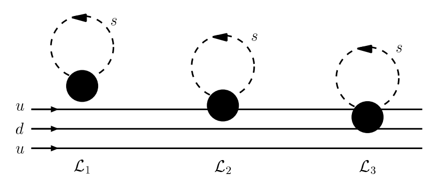

Figure 1: Instanton-induced local strangeness induced in the proton by the effective one-, two- and three-body operators in the interaction (10), namely $\mathcal{L}_1$, $\mathcal{L}_2$, and $\mathcal{L}_3$, respectively. Non-strange quarks are denoted by solid lines, and strange ones by dashed lines.

which experience the axial and trace anomaly, respectively. However, in the light of the aforementioned prevalence of model results indicating that the vector strangeness of the proton is probably significant, the vanishing vector strangeness obtained in the model of Refs. [5, 6] seemed as a drawback, a weakness of that model. Thus, when the model was further developed [9, 10], the vector strangeness and its significance was not discussed, it was hardly even mentioned. (Only the scalar strangeness for the improved model was discussed [10], and at length.)

Both in Refs. [5, 6] and Refs. [9, 10], the model is essentially the MIT bag model enriched by the presence of a dilute instanton liquid [5, 6, 7], so that it focuses on QCD-vacuum fluctuations as given by the instanton-liquid model [11, 12, 13]. However, Refs. [5, 6, 7] employed the so-called linearized approximation [14], which implies freezing the baryon radii in their original MIT values. In Refs. [9, 10] this approximation was removed: the baryon bag radii were allowed to vary in the course of parameter fitting to the masses of the baryons from the ground state octet and decuplet. (The re-fitting was performed so that the radii had to satisfy the pressure-balance condition [9, 10]. For details of the re-fitting, see Ref. [9].)

In any case, in all Refs. [5, 6, 7, 9, 10], the instanton-induced interaction of the instanton-liquid model [11, 12, 13] produces QCD-vacuum fluctuations, including presently interesting s-quark loops, schematically shown in Fig. 1. Let us denote the corresponding Lagrangian density by $\mathcal{L}_I$. The instanton-induced interaction contains the one-, two-, and three-body operators (respectively denoted by $\mathcal{L}_1$, $\mathcal{L}_2$, and $\mathcal{L}_3$ and given explicitly in Refs. [14, 5, 6]),

$$\mathcal{L}_I = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 \; . \tag{10}$$

Fig. 2 shows how an external probe couples at the vertex Γ to such an s-quark loop produced by $\mathcal{L}_I$. Various couplings are possible: $\Gamma = 1, \gamma_5, \gamma_\mu$, $\gamma_\mu\gamma_5, \sigma_{\mu\nu}$, corresponding, respectively, to the scalar, pseudoscalar, vector, axial vector, and tensor strangeness.

The instanton-induced interaction (10) contains the instanton density n as an overall factor. In the linearized approximation, the instanton density

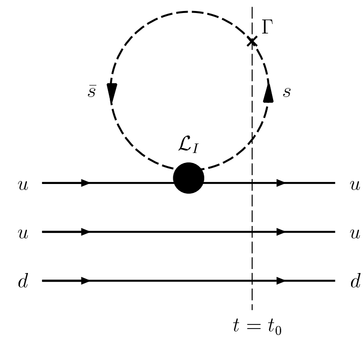

Figure 2: A non-vanishing $s\bar{s}$ component of the nucleon state found (at the moment $t = t_0$) by the probe coupled at the vertex Γ (denoted by $\times$). More precisely, this graph is that part of the proton response which arises only through one interaction $\mathcal{L}_I$. In the concrete case depicted in this figure, it is the two-body interaction $\mathcal{L}_2$.

inside the bag was found [14, 5] to be very depleted with respect to its value in the nonperturbative QCD vacuum. It is then a good approximation to keep only the first term in the perturbation series in the interaction $-\mathcal{L}_I$. Thus, the result of Ref. [5] for evaluating the nucleon-strangeness matrix element can be written as

$$\begin{aligned} \langle N| : \bar{s}\Gamma s : |N\rangle &= i \int_{-\infty}^{\infty} dt' \, \langle N_0|\hat{T} : \int d^3x \\ \times \quad & \bar{s}(\vec{x}, t_0)\Gamma s(\vec{x}, t_0) : : \int d^3y \, \mathcal{L}_I(\vec{y}, t') : |N_0\rangle \,, \end{aligned} \tag{11}$$

where : ... : denotes the normal ordering, and $|N_0\rangle$ is the model nucleon ground state composed of the non-strange valence quarks only. Note that $s(\vec{x}, t)$ denotes the strange quark field. In our model calculation, for any quark flavor q (and we are now interested only in the light flavors, $q = u, d, s$), we expand the quark fields $q(\vec{x}, t)$ in an appropriate wave-function basis $\{q_K\}$ in terms of creation $(\mathcal{U}_K^\dagger, \mathcal{D}_K^\dagger, \mathcal{S}_K^\dagger)$ and annihilation $(\mathcal{U}_K, \mathcal{D}_K, \mathcal{S}_K)$ operators of *dressed* quarks and antiquarks:

$$q(\vec{x}, t) = \sum_K \left[\mathcal{Q}_K \, q_K(\vec{x}) e^{-i\omega_K t} + \mathcal{Q}_K^{c\dagger} \, q_K^c(\vec{x}) e^{i\omega_K t} \right] \,. \tag{12}$$

Here, $q_K(\vec{r})$ denotes a model wave function of a quark of flavour q, where K stands for the set of quantum numbers labeling a model quark state.

As already said, for concrete evaluations of a nucleon-strangeness matrix element (11), Refs. [5, 6, 7, 9, 10] chose to employ the MIT bag model. With this choice, $q_K(\vec{r})$ is the solution for the quark in the K-th mode of the MIT bag. For the wavefunctions and other details of the model calculations, see especially Ref. [6].

For a concrete evaluation of a nucleon-strangeness matrix element (11), one should also specify the pertinent tensor structure of the vertex Γ – i.e., which kind of nucleon strangeness one wants to evaluate. The presently relevant case is $\Gamma = \gamma_\mu$, i.e., the case of the "vector strangeness" of the proton, as we are interested in the form factors of the vector current – see Eq. (2).

In order to calculate the contribution of the instanton-induced vector strange current inside the MIT bag, we have to identify the form factors in (9) with the Fourier-transformed vector current within the bag:

$$\begin{aligned}
&\langle N(p')| : V_\mu^s : |N(p)\rangle \\
&= \langle N(p')| : \int d^3r\, e^{-i\vec{q}_B\cdot\vec{r}}\bar{s}(\vec{r})\gamma_\mu s(\vec{r}) : |N(p)\rangle, \qquad (13)
\end{aligned}$$

using the static limit $q \to 0$. From the V_0^s component of the vector current, the electric Sachs form factor $G_E^s(q^2 = 0) = 0$ (in the leading order) in the original model employing linearized approximation [5, 6, 7], since $\langle N(p')| : V_0^s : |N(p)\rangle$ evaluated through Eq. (11) vanishes identically in the original model. Also, the calculation for the space components $\vec{V}^s$ yields the vanishing magnetic form factor, $G_M^s(0) = 0$. This implies the vanishing strange magnetic moment

$$\mu_s = F_2^s(0) = 0\ , \qquad (14)$$

Now we want to point out that this vanishing of $G_E^s(q^2 = 0)$ still holds in the improved version of the model [9, 10] not employing linearized approximation, which can be seen easily in the evaluation of Eq. (11). Obtaining the vanishing of $G_M^s(0)$ or, equivalently, $\mu_s = 0$, is not so trivial in the explicit calculation, as it happens due to a subtle cancellation among the contributions of quarks in the loop with different spin orientations (and the calculation requires careful handling of mode sums resulting from Eqs. (11) and (12) – see. Eq. (28) in Ref. [5]). In any case, one can get some non-vanishing vector strangeness only from higher orders in the instanton-induced interaction. However, such contributions would be very small and could be neglected in deriving Eq. (11) if the instanton density allowed inside the bag is sufficiently low, and this was certainly the case in the linearized approximation [5].

In retrospect, one should note that such a result of the explicit model evaluation is expected in any model on general grounds, since there is the qualitative mechanism of the suppression of the violation of the Okubo-Zweig-Iizuka rule in the vector channel [15]. It is due to the spin structure in the 'tHooft's single instanton-induced quark interaction. On the model level, removing the linearized approximation amounts (in the sense of implications on re-fitting the bag model parameters) to allowing the bag radii to vary freely, which cannot upset the aforementioned cancellation in Eq. (11).

However, removing the linearized approximation also led [10] to much larger values of the instanton density inside the MIT bag than before, in Ref. [5]). The question then arises whether Eq. (11) remains a good approximation, i.e., whether one can still consider the second and higher order terms in the instanton-induced interaction as negligibly small.

Without the linearized approximation, Ref. [10] obtained solutions where the densities n inside the bag are an order of magnitude larger than in the linearized approximation, where it was found [5] to be just $n = 0.266 \cdot 10^{-4}$ GeV^4. Nevertheless, for all acceptable phenomenological fits without the linearized approximation, Ref. [10] found that instanton densities possible inside the MIT bag, are still appreciably lower (at least by the factor of 3 or more) than n_0, the usual non-perturbative vacuum instanton density in the non-perturbative vacuum, where $n_0 \approx 1\ \mathrm{fm}^{-4} = 1.6 \cdot 10^{-3}\ \mathrm{GeV}^4$. Thus, is still justified to neglect the higher order instanton contributions and adopt the first order approximation (11). (It should be noted that Ref. [16] also estimated it could neglect higher orders in the instanton-induced interaction, although it used the full, non-depleted value of the instanton density, i.e., n_0, the instanton density appropriate for the non-perturbative QCD vacuum, in a part of the bag volume.)

4 Discussion and conclusion

At the time of publication of Ref. [5], such results on vector strangeness seemed compatible with the experimental results [17, 18] available then. However, since that time, not only other theoretical considerations, but, more importantly, preliminary announcements of more precise experimental results seemed, for a while, to point out that vector strangeness is rather large and that our approach is not suitable for treating it. The strange form factors and magnetic moment were therefore not considered in the improved version of the model beyond the linearized approximation [9, 10]. Such situation with the strange vector form factors seemed confirmed when the G0 collaboration, performing high-precision measurement at Jefferson Lab, announced large positive results for the magnetic form factor (over substantial range of momentum transfers, $0.12 \leq Q^2 \leq 1.0\ \mathrm{GeV}^2$) [19].

More recent developments, however, took a surprising turn. One may first note the recent lattice results which differ from the quoted G0 results even by the sign ($G_M^s = (-0.046 \pm 0.022)\mu_N$ [20, 21]). The most notable are of course the experimental results of the nucleon strange form factors, also obtained at Jefferson Lab but by HAPPEX collaboration, which show that the electric form factor essentially vanishes: $G_E^s(Q^2 = 0.1\,\mathrm{GeV}^2) = -0.01 \pm 0.03$ [22, 23]. This is in excellent agreement [21] with the lattice results [24] also essentially showing the vanishing of the same quantity, obtained by the same method as G_M^s [20]. Careful analyses of the methods of

extracting individual form factors revealed that it was difficult to perform an experimental separation of the individual form factors, and that it was not always clear what had been measured and what the role of theoretical input had been [21]. The proper insight has finally been gained by unifying all pertinent world data, which means the results of SAMPLE [25], A4 [26, 27], G0 [19] and HAPPEX [22, 23] collaborations, and by *joint analysis* of various form factors. For our present purposes, the most illustrative is Fig. 2 from Ref. [23], showing the data on G_E^s and G_M^s from SAMPLE, A4, G0 and HAPPEX collaborations (along with some theoretical predictions). In that plot, the ellipse shows the 95% confidence level for the possible values of G_E^s and G_M^s and indicates that the vector strangeness is not that large as people came to think previously. The best fit values are $G_E^s = -0.01 \pm 0.03$, which is perfectly consistent with zero, and $G_M^s = (+0.55 \pm 0.28)\mu_N$. While this fit thus favors nonzero values for G_M^s, we should note *i)* the suspicious sign difference with respect to the lattice results for G_M^s [20, 21], and *ii)* that the value $G_M^s = 0$ is still allowed at the 95% confidence level.

In conclusion, we have shown how the improved version [9, 10] of the model [5, 6, 7] which we used to study various aspects of the hidden nucleon strangeness, also yields the zero vector strangeness of the nucleon, namely the vanishing form factors G_E^s and G_M^s of the nucleons. While until recently this was considered wrong and an unpleasant artefact of the model, more precise measurements and more sophisticated data analysis, along with lattice QCD simulations, now show that such a vanishing vector strangeness may well be genuine, or at least that it is a good approximation. This simple model in the both variants [5, 6, 7, 9, 10] in the end turned out to be more physical than many very sophisticated models designed to produce a large vector strangeness of the nucleon.

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